

HYDRODYNAMICS AND HEAT EXCHANGE IN  
TURBULENT MOMENTUMLESS WAKES

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UDC 532.517.4:536.24

On the basis of the Reynolds equations in the approximation of the boundary layer, the article obtains the similarity solutions for the axisymmetric wake of a body with a hydrodynamic propulsion agent.

A theoretical analysis of the turbulent wakes of bodies with a hydrodynamic propulsion agent was carried out in [1], and the regularities of attenuation of the axial velocity were obtained on the assumption that the difference of the normal stresses is small. Experimental investigations [2-8] showed that the magnitude of the normal stresses plays an important part in such flows. Procedures for numerical calculation were suggested [9-12] yielding results compatible with the experimental results. The authors of [2, 9, 13-16] provided a theoretical analysis of the examined flow from different positions. The similarity solutions with one scale function of the length and different amplitude functions for different magnitudes were analyzed in detail in [16].

Below we obtain the similarity solutions for the distribution of the mean velocity over the sections of the wake and of single-point second-order momenta for velocity pulsations. We obtain the similarity laws of attenuation of the scalar magnitudes and their correlations with the velocity pulsations, and also the distributions of these magnitudes over the sections of the wake.

In a system of cylindrical coordinates, the equations describing turbulent flow behind a self-propelled body, the diffusion of temperature perturbations, in the approximation of the boundary layer, and the continuity equations are written as follows:

$$(u_\infty + u_1) \frac{\partial u_1}{\partial x} + v \frac{\partial u_1}{\partial r} + \frac{1}{r} \frac{\partial}{\partial r} (rR_{12}) + \frac{\partial}{\partial x} (R_{11} - R_{22}) = 0, \quad (1)$$

$$(u_\infty + u_1) \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial r} = -\frac{1}{r} \frac{\partial}{\partial r} (r \langle v' \theta \rangle), \quad (2)$$

$$\frac{\partial}{\partial x} (ru_1) + \frac{\partial}{\partial r} (rv) = 0. \quad (3)$$

For closing the obtained system, we write the equations for the second-order correlation momenta, written in the approximation of the boundary layer:

$$(u_\infty + u_1) \frac{\partial R_{12}}{\partial x} + v \frac{\partial R_{12}}{\partial r} - R_{12} \frac{v}{r} + R_{22} \frac{\partial u_1}{\partial r} + \frac{\partial}{\partial r} \left( \langle u' \left( v'^2 + \frac{p'}{\rho} \right) \rangle \right) = \langle \frac{p'}{\rho} \left( \frac{\partial u'}{\partial r} + \frac{\partial v'}{\partial x} \right) \rangle, \quad (4)$$

$$(u_\infty + u_1) \frac{\partial}{\partial x} (R_{11} - R_{22}) + v \frac{\partial}{\partial r} (R_{11} - R_{22}) + 2(R_{11} -$$

$$- R_{22}) \frac{\partial v}{\partial r} + 2R_{11} \left( \frac{\partial u_1}{\partial x} - \frac{\partial v}{\partial r} \right) + 2R_{12} \frac{\partial u_1}{\partial r} + \frac{\partial}{\partial r} \left( \langle v' \left( u_1'^2 - v'^2 - 2 \frac{p'}{\rho} \right) \rangle \right) = 2 \langle \frac{p'}{\rho} \left( \frac{\partial u_1'}{\partial x} - \frac{\partial v'}{\partial r} \right) \rangle - \frac{1}{r} \langle v' (u_1'^2 + v'^2) \rangle,$$

$$(u_\infty + u_1) \frac{\partial E}{\partial x} + v \frac{\partial E}{\partial r} + 2R_{12} \frac{\partial u_1}{\partial r} + 2(R_{11} - R_{22}) \frac{\partial u_1}{\partial x} = -2v \left\langle \left( \frac{\partial u_1'}{\partial x} \right)^2 \right\rangle - \frac{1}{r} \frac{\partial}{\partial r} \left( r \langle v' \left( \frac{p'}{\rho} + E \right) \rangle \right), \quad (6)$$

$$(u_\infty + u_1) \frac{\partial \langle \theta^2 \rangle}{\partial x} + v \frac{\partial \langle \theta^2 \rangle}{\partial r} = -2 \langle v' \theta \rangle \frac{\partial T}{\partial r} - \frac{1}{r} \frac{\partial}{\partial r} (r \langle v' \theta^2 \rangle) - 2a \left\langle \left( \frac{\partial \theta}{\partial r} \right)^2 \right\rangle, \quad (7)$$

A. V. Lykov Institute of Heat and Mass Transfer, Academy of Sciences of the BSSR, Minsk. Translated from *Inzhenerno-Fizicheskii Zhurnal*, Vol. 44, No. 4, pp. 640-647, April, 1983. Original article submitted November 12, 1981.

$$(u_\infty + u_1) \frac{\partial \langle v'\theta \rangle}{\partial x} + v \frac{\partial \langle v'\theta \rangle}{\partial r} + R_{22} \frac{\partial T}{\partial r} + \langle v'\theta \rangle \frac{\partial v}{\partial r} + \frac{1}{r} \frac{\partial}{\partial r} \left( r \langle \left( v'^2 + \frac{p'}{\rho} \right) \theta \rangle \right) = \frac{1}{r} \langle \frac{p'}{\rho} \frac{\partial(\theta r)}{\partial r} \rangle, \quad (8)$$

$$(u_\infty + u_1) \frac{\partial \langle u_1'\theta \rangle}{\partial x} + v \frac{\partial \langle u_1'\theta \rangle}{\partial r} + R_{11} \frac{\partial T}{\partial x} + \langle v'\theta \rangle \frac{\partial u_1}{\partial r} + \frac{1}{r} \frac{\partial}{\partial r} (r \langle u_1'v'\theta \rangle) = \langle \frac{p'}{\rho} \frac{\partial \theta}{\partial x} \rangle. \quad (9)$$

The boundary conditions are:

$$\begin{aligned} v &= \frac{\partial u_1}{\partial r} = \frac{\partial E}{\partial r} = \frac{\partial}{\partial r} (R_{11} - R_{22}) = R_{12} = \frac{\partial T}{\partial r} = \\ &= \frac{\partial \langle \theta^2 \rangle}{\partial r} = \frac{\partial \langle u'\theta \rangle}{\partial r} = \langle v'\theta \rangle = 0 \text{ for } r = 0, \\ u_1 &= E = R_{12} = R_{11} - R_{22} = T = \langle \theta^2 \rangle = \langle u'\theta \rangle = \\ &= \langle v'\theta \rangle = 0 \text{ for } r = \infty. \end{aligned} \quad (10)$$

The integral conditions are:

$$\int_0^{+\infty} (u(u - u_\infty) + R_{11} - R_{22}) r dr = 0, \quad \int_0^{+\infty} u T r dr = Q = \text{const}. \quad (11)$$

It was shown in [16] that for the given problem there exist similarity variables

$$X = x, \quad \eta = \sigma r x^{-p}. \quad (12)$$

The similarity indicator  $p$  is not determined either from the integral condition (11) or from Eqs. (1)-(9), i.e., for determining  $p$  we have to have recourse to experimental investigations or other theoretical considerations. Using the similarity variables of (12), we write the flow function  $\Psi$  introduced in accordance with the continuity equation (3), the normal stress  $R_{11}$ - $R_{22}$ , the tangential stress  $R_{12}$ , and the turbulent energy  $E$  in the form of series with decreasing powers of  $X$ :

$$\begin{aligned} \Psi &= u_\infty \sigma^{-2} [X^{4p-2} f_0(\eta) + X^{5p-4} f_{01}(\eta) + \dots], \\ R_{11} - R_{22} &= u_\infty^2 [X^{2p-2} \varphi_0(\eta) + X^{4p-4} \varphi_{01}(\eta) + \dots], \\ R_{12} &= u_\infty^2 [X^{3p-3} h_0(\eta) + X^{5p-5} h_{01}(\eta) + \dots], \\ E &= u_\infty^2 [X^{2p-2} g_0(\eta) + X^{4p-4} g_{01}(\eta) + \dots]. \end{aligned} \quad (13)$$

The system of equations (1)-(9) is open. To close it, various models [17] were suggested. We use the simplest of the possible ways of second-order closure:

$$\begin{aligned} v \left\langle \frac{\partial u_i'}{\partial x_k} \frac{\partial u_i'}{\partial x_h} \right\rangle &= c_1 E^{3/2} / \Lambda, \quad a \left\langle \frac{\partial \theta}{\partial x_k} \frac{\partial \theta}{\partial x_h} \right\rangle = s_1 E^{1/2} \langle \theta^2 \rangle / \Lambda, \\ \langle u_i' u_j' u_k' \rangle + \left\langle \frac{p'}{\rho} (u_i' \delta_{jk} + u_j' \delta_{ik}) \right\rangle &= -c_2 E^{1/2} \Lambda \frac{\partial \langle u_i' u_j' \rangle}{\partial x_k}, \\ \langle u_i' u_j' \theta \rangle + \left\langle \frac{p'}{\rho} \theta \right\rangle &= -s_2 E^{1/2} \Lambda \frac{\partial \langle u_j' \theta \rangle}{\partial x_j}, \\ \left\langle \frac{p'}{\rho} \left( \frac{\partial u_i'}{\partial x_j} + \frac{\partial u_j'}{\partial x_i} \right) \right\rangle &= -c_3 E^{1/2} \left( \langle u_i' u_j' \rangle - \delta_{ij} \frac{E}{3} \right) / \Lambda, \\ \left\langle \frac{p'}{\rho} \frac{\partial \theta}{\partial x_k} \right\rangle &= -s_3 E^{1/2} \langle u_k' \theta \rangle / \Lambda. \end{aligned} \quad (14)$$

We presume that the macroscale  $\Lambda$  is proportional to the width of the wake:

$$\Lambda = b X^p. \quad (15)$$

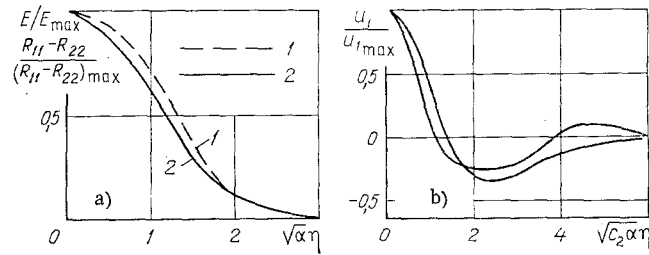


Fig. 1. Distribution: a) of normal stress (1) and of turbulent energy (2) over the section of the jet; b) of the velocity over the section of the jet.

If we substitute expressions (13) and (14) into the corresponding equations of the system (1)-(9) and equate the coefficients of equal powers of  $X$ , we obtain a system of ordinary differential equations which can be solved numerically. To simplify the obtained system, we introduce the turbulent viscosity  $\varepsilon_T$  and consider it constant across the flow:

$$R_{12} = \varepsilon_T \frac{\partial u_1}{\partial r}, \quad \varepsilon_T \propto \varepsilon E^{1/2} \Lambda. \quad (16)$$

Then, taking (16) into account, the equations for the similarity functions  $f_0$ ,  $\varphi_0$ ,  $g_0$  assume the form

$$\left(\frac{f_0'}{\eta}\right)'' + \left(\alpha c_2 + \frac{1}{\eta}\right) \left(\frac{f_0'}{\eta}\right)' + 2\alpha c_2 \left(\frac{1}{p} - 1\right) \frac{f_0'}{\eta} = \alpha c_2 \left( \left(2 - \frac{2}{p}\right) \varphi_0 - \eta \varphi_0' \right), \quad (17)$$

$$\varphi_0'' + \alpha \eta \varphi_0' - 2\alpha \varphi_0 (1 - (c_3 - b)/bp) = -\frac{2}{3\eta} g_0',$$

$$g_0'' + \left(\alpha \eta + \frac{1}{\eta}\right) g_0' - 2\alpha g_0 (1 + (c_1 - b)/bp) = 0, \quad \alpha = p/(c_2 \varepsilon b \sigma^2).$$

By the substitution  $\eta_1 = -\alpha \eta^2/2$ , the homogeneous equations of system (17) are reduced to confluent hypergeometric equations [18]. The solution of the third equation of system (17) is written via the confluent hypergeometric functions [18]:

$$g_0 = E_m \Phi \left( -1 - \frac{c_1 - b}{bp}, 1, -\alpha \eta^2/2 \right) + c \Psi \left( -1 - \frac{c_1 - b}{bp}, 1, -\alpha \eta^2/2 \right). \quad (18)$$

Only the function  $\Phi$  satisfies the boundary conditions (10). The experimental investigations [2-8] show that the profile of turbulent energy is well approximated by an exponential function, and therefore, without loss of generality, we may state that the expression  $-1 - (c_1 - b)bp$  is equal to unity on account of the selection of the constant  $c_1$ . Then expression (18) is rewritten as follows:

$$g_0 = E_m e^{-\alpha \eta^2/2}. \quad (19)$$

We substitute (19) into the second equation of system (17) and put the value of the expression  $(b - c_3)/bp - 1 = 0.5$  by suitably choosing the constant  $c_3$ , and we write the solution satisfying the boundary conditions (10):

$$\varphi_0 = B_1 e^{-\alpha \eta^2/2} + \frac{8}{3} E_m e^{-\alpha \eta^2/2} \left[ \frac{1}{8} \alpha \eta^2 + \sum_{n=2}^{\infty} \frac{(n-1)!}{n(2n-1)!} \left( \frac{1}{2} \alpha \eta^2 \right)^n \right]. \quad (20)$$

The constant  $B_1$  determines the amplitude of the normal stress on the axis of the wake. For the sake of simplification it was adopted in [15] that the profile of the normal stresses has the same form as the profile of the turbulent energy. Figure 1a shows the similarity distribution of turbulent energy (19) and of normal stress (20) over the section of the wake. It can be seen from the figure that the profile of the normal stresses has a flatter peak at the center of the wake than the profile of turbulent energy.

For the first equation of system (17), the general solution satisfying the boundary conditions (10) and the integral condition (11) is written as follows:

$$\frac{f_0'}{\eta} = B_2 \Phi \left( \frac{1}{p} - 1, 1, -\frac{1}{2} \alpha c_2 \eta^2 \right) + e^{-\alpha \eta^2/2} \sum_{n=1}^{\infty} \frac{b_n}{n!} \left( -\frac{1}{2} \alpha c_2 \eta^2 \right)^n,$$

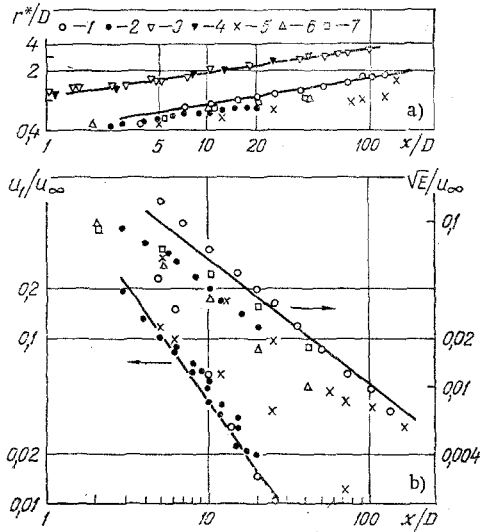


Fig. 2. Change of the width of the wake (a) and distribution of the turbulent energy and of the mean velocity along the axis of the wake (b): 1) experimental data of [2]; 2) of [3]; 3) of [4]; 4) of [5]; 5) of [6]; 6) of [7]; 7) of [8].

$$b_1 = B_1 \left( \frac{1}{p} - 1 \right), \quad b_2 = \frac{1}{2} B_1 \left( \frac{1}{c_2} + \frac{1}{p^2} - \frac{1}{c_2 p} \right) - \frac{E_m}{3c_2 p} + \frac{1}{2c_2} - \frac{1}{2p}, \quad (21)$$

$$\frac{b_n}{n!} = - \frac{b_{n-1}}{(n-1)!} \left( \frac{2pc_2 - c_2 + p}{c_2 p n^2} - \frac{1}{n} \right) - \frac{b_{n-2}}{n^2(n-2)!} \frac{1}{c^2} \left( 1 + \frac{1}{c^2} \right) + \frac{16}{3} E_m (-1)^n \left( \frac{1}{c_2} \right)^{n-1} \frac{(n-2)! (1+p-3p(n-1))}{n^2(2n-2)!}, \quad n \geq 3.$$

The constant  $B_2$  determines the value of the maximum of the velocity on the axis of the wake. The integral condition (10) is satisfied for any values of  $B_1$ ,  $B_2$ . Experimental measurements [2-7] of velocity attenuation on the axis of the wake yield the value  $p = 1/4$ , then the longitudinal velocity is

$$u_1 = \frac{1}{r} \frac{\partial \Psi}{\partial r} = u_\infty \left( \frac{f'_0}{\eta} X^{2p-2} + \frac{f'_{01}}{\eta} X^{4p-4} \right) = u_\infty \left( \frac{f'_0}{\eta} X^{-3/2} + \frac{f'_{01}}{\eta} X^{-3} \right), \quad (22)$$

i.e., the correction in the nonsimilarity term in the resolution of the velocity is small compared with the similarity solution. It can be seen from formula (21) that the scale of the transverse coordinate  $\eta$  for the velocity is proportional to  $c_2 \alpha / 2$  in distinction to the scale  $\eta$  for the turbulent energy and normal stress, where it is proportional to  $\alpha / 2$ . Figure 1b shows that in dependence on the ratio of the constants in (21), in the wake of a self-propelled body two kinds of distribution of axial velocity may be realized. In Fig. 2a the results of the calculations of the theoretical change of the width of the wake  $\propto bX^p$  for  $p = 1/4$  (solid straight line) are being compared with the available experimental data. The authors of [4, 5] studied the wake of an oscillating screen. Figure 2b presents the experimental data on the change of the maximum of turbulent energy and of the maximum of the averaged velocity along the axis of the wake; the solid line corresponds to the theoretical solution ( $p = 1/4$ ).

Let us examine the thermal wake. From the system of equations (1)-(9) and the integral condition (10) we obtain the following similarity laws:

$$T = T_\infty d_1(\eta) X^{-2p} + \dots, \quad \langle v'\theta \rangle = u_\infty T_\infty \varphi_1(\eta) X^{-p-1} + \dots, \quad (23)$$

$$\langle \theta^2 \rangle = T_\infty^2 \varphi_2(\eta) X^{-4p} + \dots, \quad \langle u'\theta \rangle = u_\infty T_\infty \varphi_3(\eta) X^{-2} + \dots$$

If we substitute the expansions (23) into (1)-(9), we obtain the system of ordinary differential equations

$$p\eta d'_1 + 2pd_1 = \sigma(\varphi'_1 + \varphi_1/\eta), \quad \beta = p/(\sigma bs_2),$$

$$\varphi''_1 + (\beta\eta + 1/\eta)\varphi'_1 + \beta(1 - (s_3 - b)bp)\varphi_1 = \frac{1}{3p} E_m \sigma \beta e^{-a\eta^2/2} d'_1, \quad (24)$$

$$\varphi''_2 + (\beta\eta + 1/\eta)\varphi'_2 + 2\beta(2 - s_1/bp)\varphi_2 = 2\varphi_1 d'_1 / bs_2.$$

Integrating the first equation of system (24), we obtain the explicit dependence of the profile of averaged temperature on the correlation  $\langle v'\theta \rangle$ :

$$\varphi_1 = \frac{p}{\sigma} d_1 \eta. \quad (25)$$

If we express function  $d_1$  from (24) and substitute it into the second and third equations of system (24), we obtain ordinary differential equations for  $\varphi_1$  and  $\varphi_2$  which can be solved numerically.

We will examine the solution of system (24) on the basis of the model for the coefficient of effective turbulent thermal diffusion  $\gamma_{T\infty} \sim \gamma \Lambda E^{1/2}$ , considering it constant across the wake:

$$\langle v'\theta \rangle = -\gamma_{T\infty} \frac{\partial T}{\partial r}. \quad (26)$$

In accordance with (25) we obtain the following differential equation for the function  $d_1$ :

$$d_1'' + d_1'(\alpha_2 \eta + 1/\eta) + 2\alpha_2 d_1 = 0, \quad \alpha_2 = p/(\gamma \sigma^2 b). \quad (27)$$

The solution of this equation is

$$d_1 = T_m e^{-\alpha_2 \eta^2/2}, \quad (28)$$

where the constant  $T_m$  is determined from the integral condition (11). We substitute expression (28) into the right-hand side of the third equation of system (24):

$$\varphi_2'' + (\beta \eta + 1/\eta) \varphi_2' + 2\beta(2 - s_1/bp) \varphi_2 = -2\gamma d_1' / s_2. \quad (29)$$

The general solution of Eq. (29) satisfying the boundary conditions (10) is expressed through the confluent hypergeometric functions [18]:

$$\varphi_2 = \theta_m \Phi \left( 4 - 2s_1/bp, 1, -\frac{1}{2} \beta \eta^2 \right) - e^{-\alpha_2 \eta^2} \sum_{n=2}^{\infty} \frac{a_n}{n!} \left( -\frac{1}{2} \beta \eta^2 \right)^n, \quad (30)$$

$$a_2 = -\gamma \alpha_2^2 T_m^2 / (s_2 \beta^2), \quad a_3 = \frac{2}{3} (1 - s_1)/(2bp) - 5\alpha_2/\beta a_2,$$

$$\frac{a_n}{n!} = \frac{a_{n-1}}{(n-1)!} \left[ \left( 2 - 2\frac{\alpha_2}{\beta} - \frac{s_1}{bp} \right) \frac{1}{n^2} - 4\frac{\alpha_2}{\beta} \left( 1 + \frac{\alpha_2}{\beta} \right) \frac{n-1}{n^2} \right] + 2\frac{\alpha_2}{\beta} \frac{a_{n-2}}{n^2(n-2)!}.$$

For  $p = 1/4$  we obtain that the attenuation rate of the averaged temperature is proportional to  $X^{-1/2}$  (for the averaged speed  $\propto X^{-3/2}$ ), i.e., the scalar magnitudes in the wake of the propulsion agent degenerate much more slowly than the vectorial ones. For the attenuation of velocity and temperature pulsations  $E \propto X^{-3/2}$ ,  $\langle \theta^2 \rangle \propto X^{-1}$ , i.e., the energy of turbulence fades much more quickly than the square of the temperature pulsations. The longitudinal correlation  $\langle u_1'\theta \rangle$  is proportional to  $X^{-2}$  and does not depend on the indicator  $p$ . The degeneration of the transverse correlation  $\langle v'\theta \rangle$  is proportional to  $X^{-5/4}$ , i.e., considerably slower than the longitudinal correlation  $\langle u_1'\theta \rangle$ .

#### NOTATION

$x, r$ , cylindrical coordinates;  $u_\infty$ , speed of the propelling agent;  $u, v$ , averaged longitudinal and transverse speed, respectively, in the wake;  $u_1$ , excess velocity in the wake;  $T$ , averaged excess temperature in the wake;  $\nu$ , kinematic viscosity;  $a$ , thermal diffusivity;  $R_{ij} = \langle u_i' u_j' \rangle$ , correlation of velocity pulsations ( $i, j = 1, 2, 3, u_1' \sim u', u_2' \sim v', u_3' \sim w'$ );  $E = \langle u_1'^2 \rangle + \langle u_2'^2 \rangle + \langle u_3'^2 \rangle$ , turbulent energy;  $p'$ , pressure pulsation;  $\rho$ , density;  $\theta$ , temperature pulsation;  $\varepsilon, \sigma, c_1, c_2, c_3, s_1, s_2, s_3, b$ , empirical constants;  $r^*$ , distance from the axis of the wake where the velocity is equal to half the maximum in the given section.

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## HEAT EXCHANGE IN ANNULAR CHANNEL WITH INTERMEDIATE HEAT CARRIER

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UDC 66.047

The article explains the results of the experimental investigation of heat exchange in an annular channel with an intermediate heat carrier. A comparison is presented with coaxial cylinders rotating in the same direction. The experimental devices are described.

A characteristic feature of all structures for drying sheet material on drums is the possibility of heat transfer from a primary heat carrier with elevated pressure and temperature to the material through intermediate heat carriers making it possible, without greatly raising the pressure in the drum cavity, substantially to increase the temperature of the drum surface, and thus to intensify the drying process. As intermediate heat carriers various high-temperature (organic and inorganic) liquids are suggested which have low vapor pressure at high temperature. At the Kaliningrad Branch of the Central Research, Project, and Design Institute for Planning Equipment of the Pulp and Paper Industry (TsNIIbummash) also a number of designs were suggested where heat transfer is effected from an inner (moving or fixed) cylindrical jacket, consisting of annular pipes and being heated by highly superheated steam, to the outer shell of the drying drum through an intermediate heat carrier that fills the closed annular space (channel) between the tubular jacket and the outer drum shell.

Inside the drying drum, heat exchange using an intermediate heat carrier proceeds similarly to heat exchange in the annular channels of coaxial cylinders where the gap between them is filled with a heat carrier that does not move axially, and the heat is transferred from the inner cylinder to the outer cooling cylinder by a heat carrier filling the space between them [1-18].

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Kaliningrad Branch of the Central Research, Project, and Design Institute for Planning Equipment of the Pulp and Paper Industry. Translated from *Inzhenerno-Fizicheskii Zhurnal*, Vol. 44, No. 4, pp. 647-651, April, 1983. Original article submitted December 30, 1981.